

Fiji Mathematics Team Competition – Zone
Form 7/ Year 13 – 2014

F7/1 A polynomial $p(x)$ has a remainder of 4 when divided by $x + 2$ and a remainder of 14 when divided by $x - 3$. What is the remainder when $p(x)$ is divided by $(x + 2)(x - 3)$?

F7/2 An army of ants is organizing a peace march across a room. If they form columns of 8 ants there are 4 left over. If they form columns of either 3 or 5 ants there are 2 left over. What is the smallest number of ants that could be in this army?

F7/3 If $\binom{n}{r}$ represents the number of combinations of n items taken r at a time, what is the value of $\sum_{r=1}^3 \binom{n}{r}$ when $n = 4$?

F7/4 If the coefficients of x^7 and x^8 in $\left(2 + \frac{x}{3}\right)^n$ are equal, then what is the value of n ?

F7/5 If a and b are positive numbers, and (x, y) is a point on the curve $ax^2 + by^2 = ab$, what is the largest possible value of xy ?

F7/6 Let the number c be defined as follows.

$$\log_c(10) + \log_c(10^2) + \cdots + \log_c(10^{10}) = 110$$

What is the value of c ?

F7/7 If $\theta = 11^\circ$, then what is the value of $(\sin \theta + \cos \theta)^2 - \sin(2\theta)$?

F7/8 There are 29 people in a room. Of these, 11 speak French, 24 speak English and 3 speak neither French nor English. How many people in the room speak both French and English?

F7/9 A drawer contains 64 socks. Each sock is one of 8 colours, and there are 8 socks of each colour. If the socks in the drawer are thoroughly mixed and you randomly choose two of the socks, then what is the probability that these two socks will have the same colour.

F7/10 Bob starts from the east end and Jane from the west end of a swimming pool, and both swim two lengths of the pool at constant rates. They pass each other twice, each time going in opposite directions. The first time they pass they are 20 feet from the east end, and the second time they are 18 feet from the west end. Assuming that each made an instantaneous turn when they reached an end of the pool, how long is the pool, in feet?

F7/11 A tank has three independent inlet pipes, A , B , and C . A and B will fill the tank in z minutes; A and C will fill the tank in y minutes; and B and C will fill the tank in x minutes. How long will it take for pipe A alone to fill it?

F7/12 Suppose that $f(x)$ is a function from the real numbers to the real numbers such that $f(x + f(x)) = 4f(x)$ and $f(1) = 4$. What is the value of $f(5)$?

F7/13 Find an equation for the line that is normal to the curve $y = x^3 - 2x + 1$ at the point $(0, 1)$.

F7/14 In January of the year 2005, I was one more than eleven times as old as my son William. In January of 2014, I was seven more than three times as old as him. How old was my son in January of 2005?

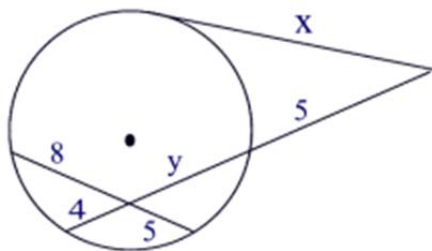
F7/15 Working together, Bill and Tom painted a fence in 8 hours. Last year, Tom painted the fence by himself. The year before, Bill painted it by himself, but took 12 hours less than Tom took. How long Tom take, when he painted alone?

F7/16 A teacher teaches 8 students how to fold an origami model. Each of these students goes on to teach 8 students of their own how to fold the same model. If this teaching process goes on for 6 generations, how many people in total will know how to fold the origami model?

F7/17 Your boss says that his wife has put an 18×51 foot garden in along the whole back end of their back yard. He says that this has reduced the back-yard lawn area by 24%. What are the dimensions of the remaining lawn area?

F7/18 Let Matrix $A = \begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix}$, Find A^{-1} .

F7/19 Given circle with tangent, secant and chord, Find the value of x.



F7/20 If 7 distinct fair 6-sided dice are rolled at the same time, what is the probability that the sum will be 10?

TIE BREAKER

F7/21 Thirty-six students took the ACT, with a mean score of 25.5. The boys had a mean score of 23.5, while the girls had a mean score of 28. How many girls were in the group?

F7/22 Several people started with \$400 each, and played a game with the following unusual rules. Each player pays \$10 to the house at the beginning of each round. During each round, one active player is declared the loser, and he distributes all of his money in equal amounts to the remaining players. The loser must then leave, but all of the other players go on to the next round. The game is over as soon as only one player remains. At the end of the game, the surviving player was surprised to discover that he had exactly \$400, equaling his starting amount. How many players were there at the beginning?
